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Examiners' Report

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Summary

The paper was well received overall, with all questions providing some access at all grade boundaries. There were not many blank responses, and most were able to access at least the early parts of question 8, indicating that candidates were able to answer the paper in the time allowed. Question 7 proved the most challenging, being the only question with a mean of under half the marks, yet the question was not unusual for the topic, perhaps indicating this was an area that candidates were underprepared for due to the pandemic. Questions 6 and 8 provided discrimination at the high grade boundaries, with question 6 in particular providing something a bit different from the usual type of method of differences questions.

Structure style and clarity varied widely in the solutions offered. Standard formulae were generally not quoted before being applied and some of those used were clearly wrong. The use of the occasional diagram would help clarify what the objective was, for instance in question 1.

Question 1

With a mean score of 5.4 out of 7 and about 50% of candidates able to score full marks, this provided a suitable opening question for the paper. Over 85% score 4 or more marks, and part (a) proved more problematic than part (b) overall.

In part (a) the method for the modulus was correctly demonstrated by the vast majority of candidates, only very few instances of incorrect method being seen. It was rare not to see the value of 8 correctly stated or calculated, although some did give −8 as a modulus.

Knowing that the tangent was required to obtain the argument was shown by most, but this alone was not enough to access the second method mark. Most did attempt the correct ratio, though as usual there were a few candidates who had the fraction the wrong way. Much more troublesome was obtaining an angle in the correct quadrant, with many giving an argument of $\frac{\pi}{3}$. Some did realise the correct quadrant but did not pay heed to the domain for θ, losing the final A. Also there were many cases where the correct angle was found, but the final answer was not given in the form specified, with $8\left(\cos\frac{2\pi}{3} - \mathrm{i}\sin\frac{2\pi}{3}\right)$ being given on numerous occasions.

Again in part (b), the method required was demonstrated by most candidates and only a minority failed to score both method marks. Of those who did not, cubing the modulus and tripling the argument was the main error.

The follow through first accuracy mark also meant there was good access for students who had obtained an angle in the wrong quadrant in part (a), and most did proceed to find two correct follow through answers, though some did stop at just one solution. Missing the i in the index was a common cause for failing to gain either accuracy mark, while a few did forget to divide the argument by 3, and there were some candidates who cube rooted twice.

To obtain full marks required all the root and no extra, and it was rare for extra solutions to be given. Most candidates who scored full marks in (a) went on to achieve full marks overall. However, notational ambiguities were often seen, but condoned, with answers such as $e^{\mathrm{i}-\frac{2\pi}{9}}$ being see frequently.

Question 2

Another accessible opening question, with mean score 4.5 out of 6, but proved to be an early discriminator as many did not know how to choose a suitable particular integral when the natural one was part of the complementary function. The modal mark was full marks, scored by just under 50% of candidates, with 3 marks scored by about another 25%. The typical score profile for these latter was M1A1B0M1A0B0, correctly finding the complementary function, and showing the knowledge of needing to differentiate a particular integral twice and substitute to try and find the constant, but being unable to do so due to the choice of particular integral.

The first two marks for the complementary function were score by most candidates. Indeed, less than 5% scored less than 2 marks. The choice of particular integral, though, was not done as well as might be hoped. Mostly candidates did not even take the complementary function into consideration and chose $y = \lambda e^{3x}$. Usually these ended up grinding to a halt when they could not find λ, or decided the constant was zero, thus being unable to score the final mark. Others realised that this was not a suitable choice for the particular integral but did not know what should be chose instead with $e^{\lambda x}$ or $xe^{\lambda x}$ being fairly common, while some chose a form without a constant to evaluate at all, such as $2e^{3x}$. The latter of these were unable to score any remaining marks.

Most candidates did then go on to differentiate their expressions, usually correctly, and substitute their changed values into the left-hand side of the differential equation to gain the second method. However, those with incorrect particular integrals often made little progress in solving, though a few did realize the error and try another. Those who tried λe^{3x} often decided $\lambda=0$ and so had no particular integral. This may suggest that many have learnt a 'rote method' for this type of differential equation but have little appreciation of what a complementary function actually is. Some instead had "constants" coming out as functions, which was not permitted the final mark, but a few did obtain the follow through on offer for finding a constant in an incorrect particular integral and knowing to add the two functions. There were a few who omitted the $y = \ldots$ and so lost the final mark on those grounds. Candidates should make sure they are careful to give a complete answer.

Question 3

Another question that provided good access for all grades in part (a), but discriminated well in part (b). The mean score was 7 out of 11, with less than 7.5% scoring 3 or fewer marks, but less than 2% obtained full marks, the final A mark in part (a) being particularly demanding. The most common scores wer3 6 or 8 out of 11.

Part (a) of the question was answered well by most candidates with the first four marks being scored by most. They were able to put the correct equations equal and find the required quadratic or cubic. Most solved the quadratic/cubic, often by calculator with just the roots stated, and found the roots and went onto find the y value for P as well. The algebra in this part was very good overall, with only a few making slips or solving the wrong equation. Also only very few failed to attempt any algebra, and stated solutions before reaching a polynomial, though such was seen occasional and candidates need to be aware of the instruction to use algebra.

The final mark of part (a), however, proved largely inaccessible. It was disappointing that very few candidates tried to show why P was a point where the curves touch rather than cross. It appears many either did not pick up on the word "touch", or were so determined to find P, they forgot to show that (or even state) at P there was a repeated root. Only very few candidates gained this mark, so it appears most assumed by finding P, they were showing that the curves touched at this point, perhaps misunderstanding, or at least not appreciating what it means to touch.

Part (b) did discriminate well. Most candidates realised that they needed to find the other intersection of the two curves, but not all managed to set up a correct equation for $x < 0$. Some used $-\dfrac{4x}{4-x}$ while other attempted to make $|x|$ the subject of an equation first, before attempting to consider the separate cases, with varying degrees of success. It is also noteworthy that many students repeated the same work from part (a) to find the critical value for $x > 0$ afresh, not appreciating how the question was set up to lead them through.

The candidates who did set up a correct equation (either directly or via making $|x|$ the subject) usually found the roots of this quadratic correctly, and often by calculator. However, the answer $2-\sqrt{10}$ was often seen, presumably a miscopy (sometimes a correct earlier version had been seen other times was all that was given).

There were also many candidates who did not identify that 0, +4 and −4 were also critical values that needed to be included when considering the range required. Some identified just one or two of these, others did not identify any, just using the results from the solutions to their equations.

Most candidates were able to correctly identify two ranges, usually this was the two extreme ranges, $x < 2-2\sqrt{10}$ and $x > 6$, gaining them two of the three marks available for identifying the range. However, it was clear that many did not use the graph to help them identify ranges, with annotation of the graph rare, and simply too the outsides of their extreme ranges, failing to spot that there were also intervals in between where the inequality holds. Candidates would be well advised to make full use of a sketch if given in a question for identifying suitable regions and be aware of any asymptotes where the nature of the graphs change as critical values in forming the solution.

Question 4

With a mean score of 7 out of 10, this question proved more accessible for the lower grade ranges than question 3, and less discriminating overall. Two third of candidates scored 7 or more marks for this question with over 25% scoring full marks, so as an early question on the paper it proved effective.

The sketch required in part (a) worked well, with the grid given to aid candidates being different. The gird gave good guidelines for which markers were able to judge the accuracy of the sketch, and most candidates were able to gain the method and the B mark. The accuracy was much more variable. Common errors were: an incorrect number of loops, maximum and minimum points not within tolerance of the relevant circles or straying too far from the required quadrant, only sketching half of graph, the circle not being plotted on r=1 and sometime $r = 1$ was plotted as a line and not a circle.

Part (b) was generally well done with many fully correct and succinct answers and a variety of methods in terms of the symmetries of the graph used to formulate the final answer. Errors in the sin terms when integrating occasionally cost the two accuracy marks, but the overall method for the integral was achieved by the majority. Finding polar areas seems to be a well-versed subject for candidates.

Where errors were made, usually it involved incorrect coefficients from squaring, or an incorrect simplification following correct use of identity, or occasionally sign errors in the identity, leading to incorrect integration, usually losing just the accuracy marks. However, in some case either no identity was used, with $\int r\,\mathrm{d}\theta$ attempted or squaring to only two terms was occasionally seen, as was expanding $\cos^2 6\theta$ to $\frac{1}{2}\left(1+\cos 2\theta\right)$. In such cases the dM mark was also lost.

As noted a variety of options for the limits was used. Usually a correct combination of limits and appropriate multiple was seen, but incorrect limits losing the dM1 mark for incorrect strategy did happen occasionally. Integrating from 0 to 2π was popular, as was integrating between 0 and π and doubling, or 0 and $\frac{\pi}{2}$ (or another quadrant) before quadrupling. Much less common, but also seen, was a narrow range, such as 0 to $\frac{\pi}{3}$ before multiplying through by 6.

Most solutions, including weaker students, managed to calculate the area of the circle $r = 1$ and subtract in some form. Usually this was done as a separate circle and subtracted at the end, occasionally as a sector with subtraction before scaling up. Integration proved a popular method rather than using the formula πr^2 (or equivalent for a sector) and a few candidates did this in combination with the outer curve using a "difference of curves" approach.

Question 5

Another question which the majority of candidates made good progress with, with a mean score of 6 out of 8 and over 40% fully correct responses, and over 75% scoring 5 or more marks. An expected and well practiced topic, even lower grade candidates were able to score well.

In part (a) most of the students found $\frac{dy}{dx}$ correctly, and if they did not little progress was generally made in the question as a whole. A few candidates did not apply the chain rule correctly and therefore did not multiply their answer by $\frac{1}{x}$ usually meaning only marks in (b) would be available. A few solutions started with the function $(4 + \ln x)^{-1/2}$ before attempting differentiation.

Finding $\frac{d^2y}{dx^2}$ correctly was less successful than finding $\frac{dy}{dx}$ but most were able to achieve a correct form. However, missing on of the $\frac{1}{x}$ terms did happen occasionally. Most candidate applied the product rule approach, making finding the denominator harder, but attempts at the quotient rule, or some combination of both methods were used.

Many candidates were then able to simplify their second derivative to get the correct denominator required, though for some candidates this was a challenging task and there were sometimes errors in coefficients and signs. Many jumped straight to the given answer without reaching an appropriate intermediate form but should be aware that "show that" questions required suitable evidence to be seen before reaching the given answer to obtain full marks. The alternative approach via implicit differentiation was rarely seen, but when attempted was usually completed successfully.

Part (b) was usually answered well with both accuracy marks gained and the most common error by far being a sign error when finding the value of the second derivative. There were, however, a number of candidates who did not quote a correct formula and did not state the values of the derivatives, but simply gave a solution. This is not to be advised as it makes the method marks unclear if the answer is incorrect and so risks losing method marks that might otherwise have been gained.

Most candidates began correctly by finding the values of y, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1$ for the first method mark, though in some instances this was only implied by answer substituted into the formula. Most then applied the correct Taylor's theorem with these values, gaining a correct expression for y. There were a minority of candidates who lost the method mark for Taylor's theorem by not including the 2! in the x^2 term, the most common reason for loss of the second method mark.

A few candidates applied Maclaurin Series instead of Taylor series, despite the question asking for the answer in ascending powers of $(x - 1)$, and many expanded the brackets, though such subsequent working was ignored if a correct expansion had initially been given. Those who had not given a correct simplified form first, though, forfeited the accuracy mark for this.

Question 6

The method of differences is an expected topic on this paper, but candidates are used to applying it to a partial fraction type expression, and the different nature of the function here clearly put off some of the weaker candidates, despite the method being laid out in the question. The mean of 6 out of 11 shows a ramping of difficulty occurred in this question, probably due to the unexpected appearance of a Pure 3 trigonometric proof at the start. Most students could score the 2 marks in (b), but this was sometimes all that was attempted, and 2 marks was the modal score, though 6 and 8 marks were not far behind in frequency.

Part (a) proved to be a surprisingly difficult proof for many candidates and many did not attempt it all. Indeed it was probably the most challenging question on the paper. The synoptic aspect, being Pure 3 work, together with the general preparedness levels due to the current pandemic, no doubt go a long way to explaining this.

There were relatively few coherent responses to this part, with many simply attempting to substitute x and y into the formula but make no actual progress with a proof. Only a minority even attempted to apply a correct compound angle formula to the question, and even then they often failed to apply it with correct suitable consistent variables. It was not uncommon to see $\tan(x-y)=\tan x-\tan y$ or even $\tan(x-y)=\frac{\tan x}{\tan y}$ or other spurious identities as candidates tried to piece together something that looked plausible. However, some did manage to demonstrate the tan/arctan process with a suitable partial identity to gain the method.

Those who did identify a suitable identity with correct consistent variables usually made good progress in applying it to the question by taking tan or arctan appropriately. Unfortunately there were a number of otherwise good solutions which were incomplete by failing to give a suitable conclusion making the identity on the paper clear, but instead use "RHS=" type shorthand.

Candidates found part (b) to be a very easy two marks, giving at least some access in this question to most candidates. Though many simplified the numerator without showing the difference, candidates were aware enough to show the expanded denominator to give the sufficient evidence of the proof. Occasional errors with the placement of the square in the denominator were the usual reasons for losing the A mark after scoring the M mark.

A few candidates did use the unusual and slightly less straightforward approach of starting by setting $A-B=2$ and $1+AB=(1+r)^2$ and then showing that A would need to be $r+2$ and that B would need to be r, which was acceptable as a method since the key reason for parts (a) and (b) was to lead in to part (c).

Unfortunately, many candidates did not even attempt the latter parts of this question, but for those who did part (c) was generally well approached, applying the method of difference well. Though a few did not apply the results to the summation, instead substituting into the given summation and getting nowhere with it, most were able to see the connection and apply the identities from (a) and (b) and identify the differences required. The method of differences was well understood and the majority of candidates proceeded to list more at least the minimum number of terms required, often more, to clearly identify those that would cancel. Failing to list sufficient terms was not overly costly, with just the first A mark withheld if that was all that was not shown, but candidates would be well advised to clearly demonstrate all non-cancelling and at least one pair of cancelling terms in any such question.

A few did lose a mark for never showing the arctan(1), but going straight for $\frac{\pi}{4}$, while some thought that arctan(1) was $\frac{\pi}{2}$. But many were able to identify the correct terms and provide a correct solution for this part. It was good to see those who attempted were able to apply the differences method on a context slightly different to examples they are used to.

For part (d) the response was more mixed with a lot of candidates, who had performed well on the earlier parts of the question, unable to spot that arctan(n+k) needed to be replaced by $\frac{\pi}{2}$ for the long term behaviour. Almost all those that did, however, went on to get the fully correct answer. However, many did not know where to start, or assumed the arctan terms became π or 0.

Question 7

For a standard F2 question on transformations of the complex plane this question proved to be the most discriminating question overall on the paper with a mean score of just under 4 out of 8. However, the modal score was full marks, achieved by over 25% of candidates, with 0 (23%) and 1 (15%) the next most common scores, with uniform distribution between. This shows that candidates could basically either do this question, or not even get started. Perhaps again preparation due to the pandemic had an effect here.

The anticipated method for part (a) was not evidenced by many (and likewise alternatives to the main method of (b) were rare), with an approach similar to that required in part (b) favoured, the majority making z the subject as a first step. Some got this far but did not substitute $u + \mathrm{i}v$ into the equation so scored no marks. Others did not even rearrange but tried substituting $x + \mathrm{i}y$ for z and usually ground to a halt.

However, most did attempt the method of making z the subject and substituting $w = u + \mathrm{i}v$ in either (a) or (b). Credit was given for the first mark in (b) wherever a suitable method was seen and was often the only mark scored. Though many did use the conjugate of the denominator to attempt Cartesian form, not all went on to extract the real or imaginary parts and set to zero to score the relevant method marks for (a) and/or (b). Instead attempting to set $|z| = 1$ was a common approach, even though this had nothing to do with the question – candidates perhaps attempting a rote method learned for finding the image of a circle rather than a line.

Of those who did show an appreciation for the method, mixing up the real and complex parts was fairly common, but credit was allowed for the working with an equation of the correct form for the latter marks in (b) so some progress could still be made. Although the algebra in reaching the equation for z in terms of w was generally good, errors in multiplying out brackets sometimes meant their real part gave a circle equation and imaginary part gave a line.

For the candidates who succeeded in setting the real part to zero from a correct equation, most went on to form the equation of a line, but not always successfully reaching $u = v$ or equivalent, with $u = v \pm 4$ being common due to a sign error expanding.

For the candidates who succeeded in setting the imaginary part to zero from a correct equation, most went on to obtain a correct circle equation and once candidates had reached $u^2 + v^2 - 3u - 3v + 4 = 0$ it was extremely rare that the correct centre and radius were not obtained. Errors tended to be in reaching this equation in the first place, when rearranging or when expanding the brackets.

While other approaches were less common, the expected route in (a) was seen reasonably frequently, though candidates sometimes stopped at reaching a Cartesian form without extracting the equation, unaware of how to spot the link between real and imaginary part. Other approaches to (a), such as finding two image points and forming the line through them, were seldom seen, but did occur.

For part (b) likewise use of the method of ALT1 was not common but frequently seen, usually getting no further than the first method mark. While other variations were again very rare but Alt 3 was seen very occasionally.

Question 8

Although most candidates were able to make good progress through most parts of this question a few did stop after part (a). Perhaps for some this may have been about time, for others it may have been because they could not see how to apply an integrating factor to the question, which was clearly expected by many. The mean mark came out at just over 7 out of 14, with modal score of 4 scored by about 15%, generally being those from part (a).

Part (a) was an expected topic and relatively standard. It was well answered by the majority of candidates although some, in attempting to replace $\frac{dy}{dx}$ by a function of v and x, chose to differentiate the given relation by v first and then use the chain rule to obtain $\frac{dy}{dx}$. This, although correct, displayed a lack of clarity in their thinking. Some others tried differentiating with respect to y first, and these were less successful. There were, of course, also several poor attempts that failed to obtain a suitable derivative, but in such instances some credit for attempting the substitution, or dealing with the $2yx(y-4x)$, was able to be gained.

Though many spotted the difference of squares after substituting, many instead fully expanded before cancelling terms, while some work with the expressions in y and rearranged to identify $(y-2x)^2$ before substituting.

Part (b) should have proved a straightforward question involving separation of variables, and working from a given equation did not even require correct work from part (a) to access it. It did not always work out this way, however, as a considerable number of candidates tried to treat this as a question on integrating factors and hence made no progress. The expectation that an integrating factor would be needed belies an effort to work by rote on a question without taking on board the question on its own merits. Since an integrating factor was not possible, candidates who attempted this approach did not make much more progress as they often did find a solution, though those who did produce an equation for v were able to score some marks in (c) and (d).

Of those that separated the variables correctly most were able to succeed in finding the equation for v in terms of x, though several failed to treat the constant of integration correctly, giving $v=\frac{1}{x^2}+c$, while others failed to include a constant of integration at all. A correct treatment of the constant of integration is vital in this type of question. Other errors made in this part usually involved incorrect integration of the $\frac{1}{v^2}$ term, resulting in a log term or $-\frac{1}{v^3}$ or similar.

As long as an equation in part (b) had been reached, part (c) was usually successfully completed, being a routine follow through mark

Part (d) was a bit more problematic. Only a few candidates were able to score all 5 marks, with the sketch providing a challenge for the most able.

The first mark was accessible as long as candidates had an equation involving a constant, but the second required a correct equation from part (c) in order to be awarded. There were quite a few who used $x = 1$, $y = -1$, and so lost this mark.

Sketching the curve was really a challenging task and some candidates did not even attempt it. Even those with a graphical calculator who were able to ascertain the shape from it were not always successful in showing all the details in their own sketch. Many manage to draw the part in the first quadrant but not the part on the left of y axis but there were several completely correct solutions with some candidates also including the asymptote at $y=2x$ although this was not demanded by the question. The common incorrect curve equation

$y = \dfrac{1}{x^2 - \dfrac{4}{3}}$ arising from use of $(1,-1)$, as well as other similar incorrect equations, were able to score the M and B marks for showing the correct asymptotes for their curve, and this was usually done correctly. Most correctly labelled the equation(s) of vertical asymptote(s) for their curve.

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